First of all, we thank you for considering the Friedrich Schiller University Jena, and in particular this master program in economics, for your future academic education! We realise that all students who join our master program make a substantial investment. We offer an interesting and rewarding study program for students with a proper background.

This self-test should help you to find out whether your skills and interests are a good match for our study program. It is, hence, in your own interest to do this test within the set time, without the help of friends, books, etc. You will find the solutions at the end of the test. Please consult these solutions after you have answered all questions. Only if you solve the self-test under these conditions, you can expect to get an informative and realistic feedback.

So, when you take this test, please make sure that you have **60-70 minutes** where you can work without being disturbed.

It is a **Multiple Choice** test. Indicate correct answers e.g. like this:

\[ 1a: \ldots, \quad 1b: \ldots, \quad 1c: \ldots \]

For each question only one answer is correct.

After the test, compare your answers with the solutions on the last page. Note that some answers give 1 point, and some give 2 points. Sum up your points:

<table>
<thead>
<tr>
<th>( \sum ) points</th>
<th>Recommendation</th>
</tr>
</thead>
<tbody>
<tr>
<td>45 – 60</td>
<td>Well done! We highly appreciate your application.</td>
</tr>
<tr>
<td>30 – 45</td>
<td>You could apply for the program but it will be quite demanding for you. You probably will need some additional help from our teachers.</td>
</tr>
<tr>
<td>&lt; 30</td>
<td>Please reconsider whether this is really a suitable program for you. Perhaps it might be better to choose another option, or to take some more basic courses in the relevant areas before you apply.</td>
</tr>
</tbody>
</table>

Be careful if you needed significantly more time than 70 minutes. Also be careful if you have less than 50% of the points in the areas *mathematics* and *statistics*. 
Check the time!
The test starts on the next page.
Part I: Mathematics

1. Let \( f(x) = \ln x \).
   Calculate \( f'(x) = \begin{cases} 1a: & x, \quad 1b: & 1/x, \quad 1c: & 1/\ln(x), \quad 1d: & \exp(x) \end{cases} \)
   Calculate \( f''(x) = \begin{cases} 1e: & 1/2x, \quad 1f: & 1/x^2, \quad 1g: & -1/x^2, \quad 1h: & \exp(x) \end{cases} \)

2. Let \( x(p) = p^{-\alpha} \).
   Calculate \( \eta = \frac{dx}{dp} \frac{\partial \eta}{\partial x} = \begin{cases} 2a: & \alpha, \quad 2b: & p^\alpha, \quad 2c: & 1/\alpha, \quad 2d: & -\alpha \end{cases} \)

3. Given the problem: \( \max_{x_1, x_2} u(x_1, x_2) \) subject to \( p \cdot g(x_1) + q \cdot x_2 = Y \).
   What belongs to the First Order Conditions for a maximum (Lagrange method)?
   \begin{align*}
   3a: & \frac{\partial u}{\partial x_1} - p \frac{\partial g}{\partial x_1} = 0, \quad \frac{\partial u}{\partial x_2} - q = 0 \\
   3b: & \frac{\partial u}{\partial x_1} - \lambda \frac{\partial q}{\partial x_1} = 0, \quad \frac{\partial u}{\partial x_2} - \lambda q = 0 \\
   3c: & \frac{\partial u}{\partial x_1} - \lambda \frac{\partial q}{\partial x_1} = 0, \quad \frac{\partial u}{\partial x_2} + \lambda q = 0 \\
   3d: & \frac{\partial u}{\partial x_1} - p \frac{\partial g}{\partial x_1} = Y, \quad \frac{\partial u}{\partial x_2} - q = Y \\
   \end{align*}

4. Given the matrix \( A = \begin{pmatrix} x & 5 \\ -y & 0 \end{pmatrix} \) and the vector \( b = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \).
   Calculate \( Ab = \begin{cases} 4a: & (2x, -2y), \quad 4b: & \begin{pmatrix} 2x \\ -2y \end{pmatrix}, \quad 4c: & \begin{pmatrix} 0 \\ -5y \end{pmatrix}, \quad 4d: & \begin{pmatrix} 2x & 10 \\ 0 & 0 \end{pmatrix} \end{cases} \)
   Calculate \( \det(A) = \begin{cases} 4e: & 5y, \quad 4f: & 5xy, \quad 4g: & 5x - y, \quad 4h: & 5x \end{cases} \)

5. Let \( g_{x} \) denote the growth rate of variable \( x(t) = \exp(g_{x} \cdot t) \) (continuous time).
   Write the equation \( \frac{M(t)}{V(t)} \cdot V(t) = Y(t) \) in growth rates:
   \begin{align*}
   5a: & \frac{gM + gV}{gp} = gy, \quad 5b: \quad gM - gp + gV = gy, \\
   5c: & gM + gp + gV = gy, \quad 5d: \quad \frac{gM}{gp} + gV = gy \\
   \end{align*}

6. Calculate the solution \((Y^*, i^*)\) of the following linear system:
   \[ Y = 100 - 2i, \quad i = -10 + 0.1Y \]
   \begin{align*}
   6a: & Y^* = 100, i^* = 10, \quad 6b: Y^* = 10, i^* = 2, \\
   6c: & Y^* = 120, i^* = 0, \quad 6d: Y^* = 100, i^* = 0 \\
   \end{align*}

7. Given \( f(x) \). What is the (first order) Taylor approximation of this function at point \( x^* = 1 \):
   \begin{align*}
   7a: & \quad T_1 f(x, 1) = f(1) - f'(x)(x - 1), \quad 7b: \quad T_1 f(x, 1) = f(1) + f'(1)(1 - x), \\
   7c: & \quad T_1 f(x, 1) = f(1) + f'(1)(1 - x), \quad 7d: \quad T_1 f(x, 1) = f(1) - f'(x)(1 - x) \\
   \end{align*}

8. Let \( h(x) = \frac{f(x)}{g(x)} \). Calculate \( h'(x) \):
   \begin{align*}
   8a: & \quad h'(x) = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}, \quad 8b: \quad h'(x) = \frac{f'(x)g(x) - g'(x)f(x)}{g(x)f(x)}, \\
   8c: & \quad h'(x) = \frac{f'(x)}{g'(x)}, \quad 8d: \quad h'(x) = f'(x) - g'(x) \\
   \end{align*}
1. In your sample with 8 observations of a normally distributed random variable you observe a sample standard deviation of 5 and a sample mean of 21. According to your (two-sided) Null hypothesis the population mean $\mu = 15$. How do you obtain a $p$-value for the above hypothesis test? (the function $\text{qt}(x,\text{df}=n)$ calculates the quantile of the $t$-distribution at $x$ with $n$ degrees of freedom, the function $\text{pt}(x,\text{df}=n)$ calculates the distribution function, respectively).

1a: None of the following answers are correct.
1b: $\text{qt}((15-21)/(5/sqrt(8)),\text{df}=7)$
1c: $1-\text{pt}((21-15)/(5*sqrt(8)),\text{df}=7)$
1d: $\text{pt}((15-21)/(5/sqrt(8)),\text{df}=7)$
1e: $1-\text{qt}((21-15)/(5*sqrt(8)),\text{df}=7)$

2. Consider the following output:

```
One Sample t-test
data: X
t = -1.207, df = 55, p-value = 0.2326
alternative hypothesis: true mean is not equal to 30
95 percent confidence interval:
26.31195 30.91543
sample estimates:
mean of x
28.61369
```

Which of the following statements are correct?

2a: None of the following answers are correct.
2b: The Null-hypothesis $E(X) \neq 30$ can be rejected.
2c: The probability that $E(X) = 30$ is 0.2326
2d: The probability that $E(X) \neq 30$ is 0.2326
2e: If $E(X) = 28.61369$, then the probability to get a sample like $X$ or one that is even more adverse to our Null hypothesis is 0.2326.
3. A random variable follows a Poisson distribution. The density function of the Poisson distribution is $P_{\lambda}(k) = \lambda^k \cdot \exp(-\lambda)/k!$. Your sample contains the observations \{1, 2, 2\}. What is the ML estimator for $\lambda$?

<table>
<thead>
<tr>
<th>3a:</th>
<th>3b:</th>
<th>3c:</th>
<th>3d:</th>
<th>3e:</th>
</tr>
</thead>
<tbody>
<tr>
<td>other value</td>
<td>0</td>
<td>4/3</td>
<td>5/3</td>
<td>5/2</td>
</tr>
</tbody>
</table>

4. You estimate model $M_1: Y = \beta_0 + \beta_1 X_1 + \mu$. You find for model $M_1$ that $\hat{\beta}_1 = -2$ and $\hat{\sigma}_{\beta_1} = 2$. Now you add another explanatory variable $X_2$ and estimate model $M_2: Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \mu$. You know that $\beta_2 > 0$. You also know that $X_1$ and $X_2$ are positively correlated. What can one say about model $M_2$?

<table>
<thead>
<tr>
<th>4a:</th>
<th>4b:</th>
<th>4c:</th>
<th>4d:</th>
<th>4e:</th>
</tr>
</thead>
<tbody>
<tr>
<td>other value</td>
<td>$\beta_1 &lt; -2$</td>
<td>$\beta_1 &gt; -2$</td>
<td>$\beta_1 = -2$</td>
<td>$\beta_1 &gt; \beta_2$</td>
</tr>
</tbody>
</table>

5. You estimate $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \mu$. $X_1$ is a discrete variable which is 1 for females and 0 for males. You obtain estimates $\hat{\beta}_0 = 5, \hat{\beta}_1 = -3, \hat{\beta}_2 = -1$. Now you code $X_1$ differently: $X_1$ is 1 for males and 0 for females. What are the estimates based on the new coding of $X_1$?

<table>
<thead>
<tr>
<th>5a:</th>
<th>5b:</th>
<th>5c:</th>
<th>5d:</th>
<th>5e:</th>
</tr>
</thead>
<tbody>
<tr>
<td>None of the following answers are correct.</td>
<td>$\hat{\beta}_0 = 5, \hat{\beta}_1 = -3$</td>
<td>$\hat{\beta}_0 = 2, \hat{\beta}_1 = -3$</td>
<td>$\hat{\beta}_0 = 5, \hat{\beta}_1 = 3$</td>
<td>$\hat{\beta}_0 = 2, \hat{\beta}_1 = 3$</td>
</tr>
</tbody>
</table>
Part III: Basic Microeconomics

Please complete the sentences or indicate correct answers as appropriate:

1. Productivity can be measured as the ratio between _____________.
   - 1a: capital and labour
   - 1b: output and input
   - 1c: value added and employment

2. Combinations of input and output that lie ________ the production function are ____________.
   - 2a: on, efficient
   - 2b: below, impossible
   - 2c: above, inefficient

3. Along the ________________, utility is constant.
   - 3a: isoquant
   - 3b: indifference curve
   - 3c: production function
   - 3d: utility function

4. Under typical textbook assumptions, consumer surplus is higher in ____________ than in ____________.
   - 4a: Cournot duopoly, perfect competition
   - 4b: monopoly, Cournot duopoly
   - 4c: perfect competition, monopoly

5. Which of the following statements is false? For the utility function $u(x_1, x_2) = x_1^{0.5} x_2^{0.5}$, the marginal rate of substitution $\frac{dx_2}{dx_1} = -\frac{\partial u/\partial x_1}{\partial u/\partial x_2}$ is ____________
   - 5a: constant along the indifference curve.
   - 5b: equal to the slope of the budget constraint in the optimum.
   - 5c: -1 if $x_1 = x_2$.
   - 5d: the slope of the indifference curve.
Part III: Basic Microeconomics

6. Given the following cost function: \( C(q) = 10 + 0.5 \cdot q^2 \). Which of the following statements about the marginal cost function (MC), the average cost function (AC), and the average variable cost function (AVC) is correct?

6a: MC and AC do not intersect.
6b: AC are always larger than AVC.
6c: The intersection between AC and MC is in the minimum of MC.

7. Demand \( (q) \) for a homogeneous good as a function of price \( (p) \) is given by \( q(p) = 52.5 - \frac{1}{2}p \). The cost function of the only firm on that market is \( C(q) = 5q \). What is the profit maximizing price output combination of the monopolist?

7a: \( q = 50, p = 5 \)
7b: \( q = 26.25, p = 52.5 \)
7c: \( q = 25, p = 55 \)
1. Real versus nominal variables:

The following figure shows the time series of both real GDP and nominal GDP for Germany between 1994 and 2016 (in Billion Euro).

1a) What is the difference between real and nominal variables?

1ai: *Real* variables are adjusted for the change in the price level, while *nominal* variables reflect both changes in prices and quantities.

1aii: *Nominal* variables are adjusted for the change in the price level, while *real* variables reflect both changes in prices and quantities.

1aiii: *Real* variables equal their *nominal* counterparts after subtracting taxes and adding subsidies.

1aiiv: *Nominal* variables equal their *real* counterparts after subtracting taxes and adding subsidies.

1b) Which line shows real GDP? Which one shows nominal GDP?

1ibi: The solid line represents nominal GDP, the dashed line is real GDP.

1bii: The solid line represents real GDP, the dashed line is nominal GDP.

1c) Why do both time series intersect in 2010?

1ici: The financial crisis led to a sharp decrease in real GDP.

1icii: 2010 is the base year for the computation of real GDP.

1iciii: The intersection in 2010 occurs just by chance.

2a) Which of the following statements is true in context of the Phillips Curve?

- 2ai: The Phillips Curve captures the correlation between inflation and wages.
- 2aii: The Phillips Curve captures the short-run negative trade-off between inflation and unemployment.
- 2aiii: The long run Phillips Curve has a positive slope.
- 2aiv: The inflation rate depends in the long run primarily on the unemployment rate.

2b) Which statement is true regarding the adjustment process of inflation and unemployment to expansionary monetary policy? Consider the following graph and the New Keynesian Phillips Curve represented by $\pi_t = E_t \pi_{t+1} - \beta (u_t - u_N)$ to evaluate the statements.

- 2bi: Due to expansionary monetary policy, unemployment increases temporarily.
- 2bii: Through the adjustment of inflation expectations, the temporary decrease in employment is neutralized in the long run.
- 2biii: With $E_t \pi_{t+1} = \pi_t$ it follows that $u_t = u_N$, implying a vertical long run Phillips Curve.
- 2bv: Assuming that $E_t \pi_{t+1} = \alpha \pi_{t-1}$, $\alpha$ needs to be equal to 0.5 so that the long-run Phillips Curve is vertical.
1. Assume that the tax function $T(y)$ is progressive. Which of the following statements is correct?

1a: Progressivity violates the principle of vertical tax equity.

1b: The tariff $T(y) = \begin{cases} ay & \text{if } y > c \\ 0 & \text{if } y \leq c \end{cases}$ with $a, c > 0$, is progressive.

1c: Two individuals earn the same total income (100,000) over two periods. Individual 1 earns 50,000 in both periods while individual 2 earns 80,000 in period 1 and 20,000 in period 2. The total tax burden of individual 2 is larger than that of individual 1 (no discounting).

2. The demand function $x^D$ and the supply function $x^S$ are given, respectively, by $x^S = 5$ and $x^D = 3 - 2q$, where $q$ denotes the consumer price. A tax is levied on the consumer.

2a: The producer bears the full tax burden.

2b: The consumer bears the full tax burden.

2c: Both, consumer and producer, share the burden.

3. Which of the following statements about the excess burden is correct? (Excess burden, also known as the deadweight loss of taxation, is one of the economic losses that society suffers as the result of taxes or subsidies.)

3a: The excess burden increases over-proportionally with the tax rate.

3b: The excess burden is due to the income effect from taxation.

3c: With lump-sum taxes or uniform taxes, the problem of the excess burden is particularly severe.

4. The inverse elasticity rule is obtained by choosing the set of commodity taxes that maximize the welfare of a single consumer subject to the government tax requirement.

4a: This rule is an approximation that is valid only if the cross-price effects are close to unity.

4b: This rule says that the rate at which a commodity is taxed should be inversely proportional to the absolute value of its elasticity of demand.

4c: This rule describes a first-best policy of taxation.
Part V: Public Finance

5. A commodity tax $t$ is levied on a good $x$ (see the following graph). Which of the statements is correct?

5a: The reduction from $x_0$ to $x_1$ indicates the substitution effect.
5b: The total effect on $x$ is negative.
5c: The budget constraint with the tax is $B'$. 
Part VI: Theory of Economic Policy

1. What is meant by subadditivity of costs?

1a: It is less costly, if the whole amount of a commodity \((X)\) is produced by one firm instead several firms producing subsets of this amount.

1b: It is more costly, if the whole amount of a commodity \((X)\) is produced by one firm instead several firms producing subsets of this amount.

1c: It is more costly, if two commodities \((X \text{ and } Y)\) are produced together than separately.

2. What are “economies of size”?

2a: An indicator for welfare

2b: \(C(X,Y) < C(X) + C(Y)\) (with \(C(\cdot)\) as the cost function)

2c: The average costs per unit decrease with increasing output.

3. What kind of good is exclusive and rivaling in consumption?

3a: Private good, 3b: Common good, 3c: Club good, 3d: Public good

4. What kind of good is non-exclusive and non-rivaling in consumption?

4a: Private good, 4b: Common good, 4c: Club good, 4d: Public good

5. What does it mean if marginal costs of a good converge towards zero?

5a: The profit that can be made with this good increases.

5b: The optimal price of this good is also converging towards zero.

5c: The additional costs of a good are so marginal that they can be neglected.

6. What is the transformation curve all about?

6a: It indicates the production-possibility frontier in an economy with constant factor endowments.

6b: It indicates the growth of factor endowments with constant production-possibilities.

6c: It measures to which degree a country successfully mastered the transition from a centrally planned towards a market economy.

7. What are sunk costs?

7a: Irreversible costs that cannot be recovered

7b: Costs that converge to zero due to economies of scale

7c: Additional costs of international trade due to sunk ships
Part VI: Theory of Economic Policy

8. What is meant by pareto-optimal?

8a: A situation where market regulation is very low.
8b: A situation where a re-allocation of resources does not make any individual better off without making at least one other individual worse off.
8c: A situation where only the re-allocation scheme developed by Pareto can decrease the elasticity of demand.

9. Why do markets fail in case of public goods or external effects (in a standard neoclassical model)?

9a: Because there is a discrepancy between individual and social welfare.
9b: There is no market failure.
9c: Because the government regulated the market wrongly.
9d: Because the firms acted selfishly.

10. Asymmetric information violates which standard neoclassical model assumption?

10a: Full information
10b: Indivisibility of goods/production factors
10c: Atomistic market structure
10d: Full mobility of goods/production factors

11. What standard neoclassical model assumption fails when natural monopolies occur?

11a: Full information
11b: Full market transparency
11c: Atomistic market structure
11d: Market participants maximize their utility (as known as acting “rational”)

12. A country with higher exports than imports records a ...?

12a: Current account surplus
12b: Current account deficit
12c: Capital account surplus
12d: Capital account deficit

13. What does a current account surplus imply within the balance of payment system?

13a: A balanced current account
13b: A capital account deficit
13c: A capital account surplus
13d: A decrease in currency reserves
Check the time!
The solutions are provided on the next page.
Solutions:

Part I: (1 point for each)
1b, 1g, 2d, 3b, 4b, 4e, 5b, 6d, 7c, 8a

Part II: (2 points for each)
1a, 2a, 3d, 4b, 5e

Comment on 3d): \( L = \lambda^5 e^{-3\lambda}/4 \), hence \( dL/d\lambda = (3\lambda^5 - 5\lambda^4)e^{-3\lambda}/4 \), hence \( \lambda = 5/3 \) is maximum.

Comment on 4b): If \( X_1 \) and \( X_2 \) are positively correlated, then we know that \( X_2 = \xi X_1 + \epsilon \), hence, \( M_2 \) can be written as \( Y = \beta_0 + \beta_1 X_1 + \beta_2 (\xi X_1 + \epsilon) = \beta_0 + X_1 (\beta_1 + \beta_2 \xi) + \beta_2 \epsilon \).

Since \( \xi > 0 \) and \( \beta_2 > 0 \) we have that \( \beta_1 < -2 \) in \( M_2 \).

Part III: (1 point for each)
1b, 2a, 3b, 4c, 5a, 6b, 7c

Part IV: (2 points each)
1ai, 1bi, 1cii, 2aii, 2biii

Part V: (2 points each)
1c, 2a, 3a, 4b, 5b

Part VI: (1 point each)
1a, 2c, 3a, 4d, 5b, 6a, 7a, 8b, 9a, 10a, 11c, 12a, 13b

The sum of all points is 60.