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Assembly line balancing: Joint precedence graphs under high product variety

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Abstract
Previous approaches for balancing mixed-model assembly lines rely on detailed prognoses of the demand for each model to be produced on the line (model-mix). With the help of the anticipated model-mix a joint precedence graph for a virtual average model is deduced, so that the mixed-model balancing problem is reduced to the single-model case and traditional balancing approaches can be employed. Today's ever increasing product variety often impedes reliable prognoses for individual models. Instead, forecasts for the estimated occurrences of each product feature (e.g., percentage of cars with air conditioning) are merely obtainable. This paper shows how the generation of joint precedence graphs is to be altered to account for this fundamental change in information. This way, a balancing of mixed-model assembly lines which are confronted with a high degree of product variety is enabled.

Keywords: Product variety; Mixed-model assembly lines; Balancing; Joint Precedence Graphs

1 Introduction

In a *mixed-model assembly line*, setup times and costs have been reduced sufficiently enough to be ignored, so that different products can be jointly manufactured in intermixed product sequences (lot size of one) on the same line. In spite of the tremendous efforts to make production systems more versatile, this usually requires quite homogeneous production processes. As a consequence, typically, all models are variations of

Model	Bodies	Power trains	Paint-and-trim combinations	Factory-fitted options	Number of models
Fiat Punto	2	5	51	8	39,364
Renault Clio	2	10	57	9	81,588
Ford Fiesta	2	5	57	13	1,190,784
Renault Megane	2	6	52	14	3,451,968
GM Astra	4	11	83	14	27,088,176
GM Corsa	2	9	77	17	36,690,436
Ford Focus	4	11	64	19	366,901,933
VW Golf	3	16	221	26	1,999,813,504
Fiat Stilo	3	7	93	25	10,854,698,500
VW Polo	2	9	195	27	5.26E+10
Mini (BMW)	1	5	418	44	5.10E+16
BMW 3-Series	3	18	280	45	6.41E+16
Mercedes C-Class	2	16	312	59	1.13E+21
Mercedes E-Class	2	15	285	70	3.35E+24

Table 1: Number of options and models for selected European cars

the same base product and only differ in specific customizable product attributes, also referred to as *options*.

During the configuration planning of an assembly line the so called *assembly line balancing problem*(ALBP) is to be solved, which decides on the assignment of tasks and all their required resources to the workstations of the line (e.g. Baybars, 1986; Scholl and Becker, 2006; Becker and Scholl, 2006; Boysen et al., 2006a+b). For an algorithmic solution the mixed-model ALBP is usually transformed to the single model case by the use of a *joint precedence graph* (Thomopoulos, 1970; Macaskill, 1972; van Zante-de Fokkert and de Kok, 1997). Here, the model dependent processing times of tasks are averaged with regard to the estimated demand portions (probabilities) of respective models in the model-mix and are then composed to form a unique precedence graph.

The recent trends of mass-customization (Pine, 1993) and assembly-to-order (Mather, 1989) lead to a tremendously increased product variety, so that, in many fields of business, the product variety is too large to allow for considering all models and forecasting their demands explicitly. For example, many car manufacturers offer their cars in a huge number of models, which can be configured by combining the options offered. Table 1 (extracted from Pil and Holweg, 2004) shows a selection of car types produced by European car manufacturers together with the number of offered product options (divided into the four groups car bodies, power trains, paint-and-trim-combinations and factory-fitted equipment options) and the number of (theoretically) resulting models. For further information on the product variety of the car manufacturers BMW and Mercedes see Röder and Tibken (2006) as well as Meyr (2004).

Table 1 shows that the number of models exponentially grows with the number of options. This becomes obvious by assuming that each of n independent zero/one-options can be present or not in any model such that a total of 2^n models result.

Because only a small selection of these (theoretical) option combinations are actually demanded and, thus, only very few models are repeatedly assembled, there is no adequate basis for estimating future demand rates. Especially manufacturers of luxury class automobiles state that only precious few different models are sold more than once a year (Meyr, 2004). Instead, reliable estimations can be provided only for the frequency of option occurrences over all models (*option-mix*), e.g., the percentage of cars equipped with air conditioning. Moreover, a procedure for generating a joint precedence graph, which has to iterate through all possible models, suffers from the extraordinary computational requirements in this order of magnitude. Consequently, the generation of joint precedence graphs is to be altered to account for this fundamental change in information and should be based on the options and the respective option-mix.

Increasing product variety is not a phenomenon car manufacturers have to cope with exclusively (e.g., Randall and Ulrich, 2001). Although examples presented throughout this paper stem from the automobile industry, the proposed approach is highly recommendable for configuration planning of any mixed-model assembly line which is confronted with a high degree of product variety.

The remainder of the paper is structured as follows. Section 2 briefly summarizes the generation of traditional joint precedence graphs based on model-mix forecasts, whereas Section 3 describes a modified approach based on an option-mix prognosis. Section 4 compares both approaches with respect to effort and outcome. In Section 5, the relevance of the introduced approach is compared to common business practice and evaluated by a computational experiment. The insights are then summarized in Section 6.

2 Model-based generation of joint precedence graphs

The traditional generation of a joint precedence graph requires information about the individual precedence graphs $G_m = (V_m, E_m, \mathbf{t}_m)$ of each model $m \in M$. The node set V_m contains the model specific tasks, the arc set E_m reflects precedence relations (i, j) between tasks $i, j \in V_m$, and the vector of node weights $\mathbf{t}_m$ contains the processing times t_{im} of the tasks $i \in V_m$. Additionally, demands for each model throughout the planning horizon have to be estimated, so that demand portions $0 \leq P_m \leq 1$ for each model m with $\sum_{m \in M} P_m = 1$ can be determined. This is usually done by counting model occurrences in the sales database which are adjusted based on market analyses.

The joint precedence graph $G = (V, E, \bar{\mathbf{t}})$ results from the following definitions (e.g. Macaskill, 1972; van Zante-de Fokkert and de Kok, 1997):

$$V = \bigcup_{m \in M} V_m \tag{1}$$

$$\bar{t}_i = \sum_{m \in M} P_m \cdot t_{im} \qquad \forall i \in V \tag{2}$$

$$E = \bigcup_{m \in M} E_m \setminus \{\text{redundant arcs}\} \tag{3}$$

As a prerequisite for the generation of the joint node set V in equation (1), tasks which are common to different models, albeit requiring different processing times, receive a model-

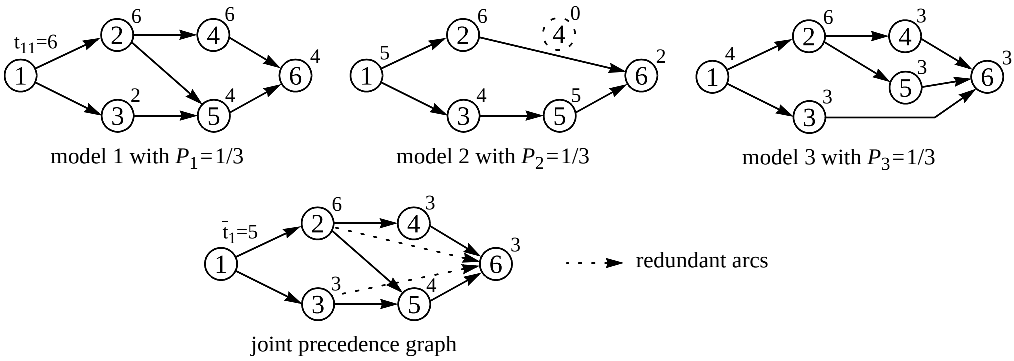

Figure 1: Joint precedence graph based on model-mix prognosis

wide consistent node number. This impedes an assignment of these tasks to different stations, which otherwise would necessitate multiple investments in required resources at each station to which a duplicate task is assigned. Tasks not required by a model receive a processing time (node weight) of 0. Thus, average processing times $\bar{t}_i$ can be simply calculated by weighting each model-specific task time t_{im} with the respective demand portion P_m of the model in equation (2). Equation (3) determines joint precedence constraints by joining the model-specific arc sets. This can lead to redundant arcs (i, j) which represent transitive precedence relationships. An arc is redundant and can thus be deleted without loss of information, if there exists another path from node i to j with more than one arc.

Further steps have to be performed, if conflicting precedence relations exist between models which lead to cycles in the joint precedence graph. To enable a unique processing sequence of tasks these cycles have to be eliminated by one of the following actions (c.f. Ahmadi and Wurgaft, 1994):

- The models have to be separated into subsets in such a manner that two or more acyclic joint precedence graphs can be formed. During physical production, this leads to setup operations which have to be performed whenever production changes from one subset of models to another.

- In order to achieve a unique task-station-assignment, cycles in the precedence graph can be eliminated by a duplication of nodes. To minimize the number of duplicated nodes and therefore reduce the danger of assigning equal tasks to different stations an optimization problem is to be solved (see Ahmadi and Wurgaft, 1994).

Figure 1 exemplifies the generation of a joint precedence graph based on a model-mix prognosis.

3 Option-based generation of joint precedence graphs

Our modified approach adopts the general procedure and the structure of the joint precedence graph regarding node set V and arc set E of the traditional approach. All tasks and their respective precedence constraints are transferred to an acyclic joint precedence graph. Cycles are resolved by node duplication. The main difference consists in determining the (expected) joint task times based on the estimated fraction (interpreted as probability) of product units containing certain options (option-mix) instead of respective forecasts for the huge number of individual models (model-mix) as discussed in Section 2.

Based on the set of all options O, the joint processing times $\bar{t}_i$ are computed for each task $i \in V$ separately by performing the following steps:

(1) Determine the set of options $O_i \subseteq O$ which require the task i to be carried out. If a task only becomes necessary if several options occur in a combined manner, O_i contains all of these options.

(2) Determine the set of option combinations $VO_i \subseteq \mathbb{P}(O_i)$ which can feasibly appear in the same model with $\mathbb{P}(O_i)$ being the power set of O_i. The single options in set O_i are temporarily replaced by their combinations in VO_i which are called *virtual options*. Note that the set VO_i also contains the empty set, if it is feasible that no option out of O_i is chosen, and all elements of O_i which can be chosen independently of other options.

(3) For each virtual option $v \in VO_i$ the respective task time $t_i(v)$ and the probability $p(v)$ are to be determined, where $p(v)$ denotes the probability of the event that all options $o \in v$ are selected and all of the remaining options $o \in O_i \setminus v$ are deselected. If task i does not have to be carried out for a certain virtual option $v \in VO_i$, task time $t_i(v)$ is set to 0.

(4) The joint task times are then computed as follows:

$$\bar{t}_i = \sum_{v \in VO_i} p(v) \cdot t_i(v) \quad \forall i \tag{4}$$

Due to the specific structure of assemble-to-order production systems, the determination of joint task times will be rather simple for the majority of tasks. Let us assume that the task set V is split up into three disjoint subsets $V^A \cup V^B \cup V^C = V$ to which a task i is assigned with regard to the number and interaction of options in set O_i as follows:

1. *Common tasks:* The tasks $i \in V^A$ have to be carried out on any model independent of the options required, i.e. $O_i = O$. Additionally, their task times t_i are identical for all option combinations. Then, the joint task times are simply given by $\bar{t}_i = t_i$ thus replacing steps (2) to (4) for all tasks $i \in V^A$.

2. *Single-option tasks:* The subset V^B contains all tasks which can be assigned to a single option occurrence exclusively. This includes all tasks which are required by a single zero/one option ($|O_i| = 1$), such as the air conditioning which may be present or not. Additionally, all tasks are covered which are required by multiple options ($|O_i| > 1$) provided that these options cannot occur in the same model. In the automobile industry, this is, e.g., the case when sunroofs are installed. The task "mounting of grommet" has to be performed (maybe at different processing times) irrespective of the exact type of sunroof, be it electric or manual. Although this task is thus required by multiple options, it can be assigned exclusively to an option in any possible model, as the options electric and manual sunroof are mutually exclusive and can never occur together in the same car. For such a single-option task i the joint processing time $\bar{t}_i$ is equal to the weighted average over all option-specific task times t_{io} in proportion to the occurrence probability p_o of the respective option o. Thus, the steps (2) to (4) are replaced by directly computing:

$$\bar{t}_i = \sum_{o \in O_i} p_o \cdot t_{io} \quad \forall i \in V^B \tag{5}$$

3. *Multiple-option tasks:* The remaining subset V^C includes any task i, whose set of options ($|O_i| > 1$) contains at least two options which can occur or not independently of each other. Such tasks are, e.g., inevitable whenever electrical components are installed in the door of a car. If electrical exterior mirrors and/or power windows are chosen, the power supply has to be made accessible by a wiring harness. This installation is thus a shared task, which becomes necessary whenever either one of the door components is chosen separately or if both options occur jointly. Moreover, task times may diverge between any of the possible option combinations. Thus, the steps (2) to (4) are to be applied to all tasks $i \in V^C$.

Remark: If the processing times of a multiple-option task are equal for all option combinations, the probability of occurrence can be adjusted by the so called *inclusion-exclusion method* attributed to Poincaré (see Jordan, 1972). Unfortunately, this does not lead to a significant simplification, because the probabilities of all possible option combinations are required nonetheless.

Example: To clarify the procedure of computing the joint precedence graph based on the option-mix, we consider a small part of a mixed-model assembly line producing cars where the above-mentioned options concerning the sunroof and the wiring harness are relevant. The structure of the joint precedence graph is illustrated in Figure 2(a).

The basic car model contains neither a sunroof nor an electrical equipment in the door. Irrespective of the option selection, the tasks 1 and 7 are required (common tasks), i.e., these tasks are members of V^A and $O_1 = O_7 = \{1, 2, 3, 4\}$. As (joint) processing times, $\bar{t}_1 = 4$ and $\bar{t}_7 = 5$ are to be regarded. As option 1 a manual sunroof or, alternatively, as option 2 an electrical sunroof can be added. Both options require the tasks 5 and 6, i.e., $O_5 = O_6 = \{1, 2\}$, but at different processing times $t_{51} = 6$ and $t_{52} = 7$ as well as

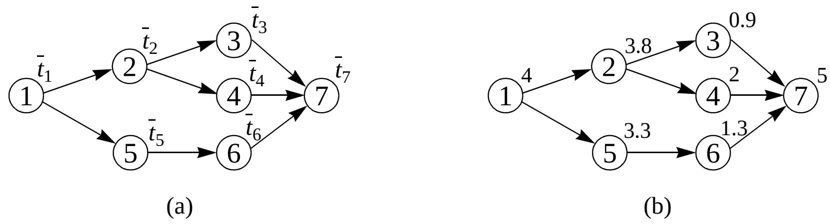

Figure 2: Joint precedence graph: structure and final graph

car ID	sunroof none	sunroof manual	sunroof electrical	electrical device in door none	electrical device in door power window	electrical device in door electrical mirror
1	-	x	-	-	x	-
2	x	-	-	-	-	x
3	-	-	x	x	-	-
4	x	-	-	-	x	x
5	-	x	-	x	-	-
6	x	-	-	-	-	x
7	-	-	x	-	x	-
8	x	-	-	-	-	x
9	-	-	x	x	-	-
10	x	-	-	-	-	x
frequency	0.5	0.2	0.3	0.3	0.3	0.5

Table 2: Extract of the sales data base as a basis of probability estimation

$t_{61} = 2$, and $t_{62} = 3$. Because both options are mutually exclusive, the tasks 5 and 6 are single-option tasks and, thus, belong to V^B. The isolated occurrence probabilities $p_1 = 0.2$ and $p_2 = 0.3$ have been estimated by extracting the respective frequencies from the sales database in Table 2. As joint processing times, we get $\bar{t}_5 = 0.2 \cdot 6 + 0.3 \cdot 7 = 3.3$ and $\bar{t}_6 = 0.2 \cdot 2 + 0.3 \cdot 3 = 1.3$ via formula (5).

Option 3 is a power window and option 4 an electrical exterior mirror. Whenever option 3 and/or option 4 are set, a wiring harness has to be installed in the door. Because these options can occur in any combination, the corresponding task 2 belongs to the set V^C. In case of option 3 the additional task 3 ($O_3 = \{3\}$) and in case of option 4 the additional task 4 ($O_4 = \{4\}$) have to be performed. Both tasks are members of the set V^B of single-option tasks. The individual occurrence probabilities estimated from the sales database in Table 2 are $p_3 = 0.3$ and $p_4 = 0.5$. The task times $t_{33} = 3$ and $t_{44} = 4$ are independent of each other such that we obtain $\bar{t}_3 = 0.3 \cdot 3 = 0.9$ and $\bar{t}_4 = 0.5 \cdot 4 = 2.0$.

In order to determine the processing time of task 2, four virtual options $VO_2 = \{\emptyset, \{3\}, \{4\}, \{3,4\}\}$ are to be regarded in step (2): The virtual option $\emptyset$ for the absence of electrical devices in the door, the virtual option $\{3\}$ for choosing the power window only, the virtual option $\{4\}$ for the electrical exterior mirror only, and the virtual option $\{3,4\}$ for the joint occurrence of both the power window and the electrical mirror. For all $v \in VO_2$, the respective occurrence probabilities $p(v)$ and task times $t_2(v)$ are to be esti-

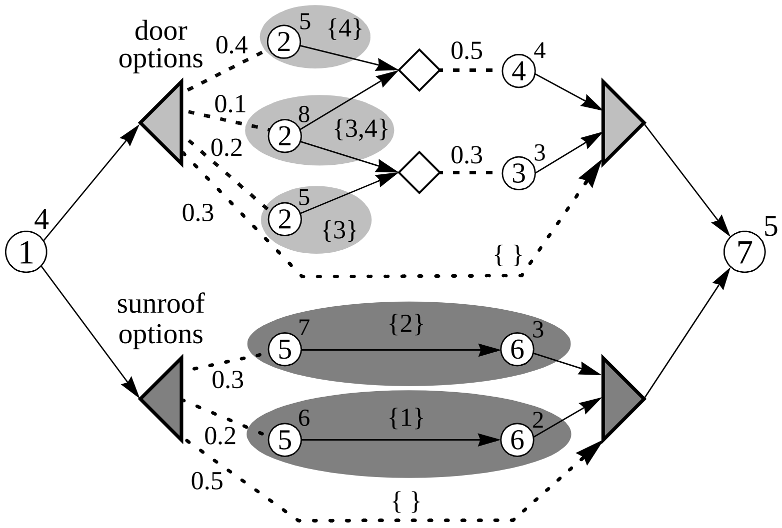

Figure 3: Option-based precedence graph

mated in step (3). Considering Table 2, we get the observed (joint) frequencies $p(\emptyset) = 0.3$, $p(\{3\}) = 0.2$, $p(\{4\}) = 0.4$, and $p(\{3,4\}) = 0.1$ as reasonable estimates. As task times, we obviously get $t_2(\emptyset) = 0$ and assume $t_2(\{3\}) = t_2(\{4\}) = 5$ and $t_2(\{3,4\}) = 8$ due to the increased amount of work if both original options 3 and 4 are combined. By applying formula (4) in step (4), we get $\bar{t}_2 = 0.3 \cdot 0 + 0.2 \cdot 5 + 0.4 \cdot 5 + 0.1 \cdot 8 = 3.8$. The resulting joint precedence graph is displayed in Figure 2(b).

The information necessary to compute the joint precedence graph is summarized in Figure 3 which represents an option-based view of the precedence graph. This graph additionally contains pairs of triangular XOR-nodes which express that the connected options are mutually exclusive. In each model, exactly one dashed arc is followed whose arc weight is given by the occurrence probability $p(v)$ of the connected (virtual) option v. So, a pair of XOR-nodes is used for the sunroof options 1 and 2 (interpreted as virtual options) and the corresponding null-option "no sunroof" and another one for the four virtual door options. The rhombical nodes allow for consolidating different paths which are common for certain tasks. The weight of the outgoing arc specifies the probability of reaching this arc. The arcs connecting a rhombus to task 3 and 4 get the weights $p_3 = p(\{3\}) + p(\{3,4\}) = 0.3$ and $p_4 = p(\{4\}) + p(\{3,4\}) = 0.5$.

Remark: The option-based precedence graph might be a useful tool even in constructing the joint precedence graph's structure. This is done by merging all copies of the same node in the option-based graph, unifying all precedence relations, and removing cycles if necessary. The joint processing time of a task i can be computed by equation (4), because the option-based graph contains a node copy of i for each virtual option $v \in VO_i$. The probability $p(v)$ is determined along the path reaching the respective node by multiplying the weights of the arcs emerging from XOR-nodes (omitting the consolidation nodes which are introduced for presentation purposes only). Notice that XOR-nodes might be

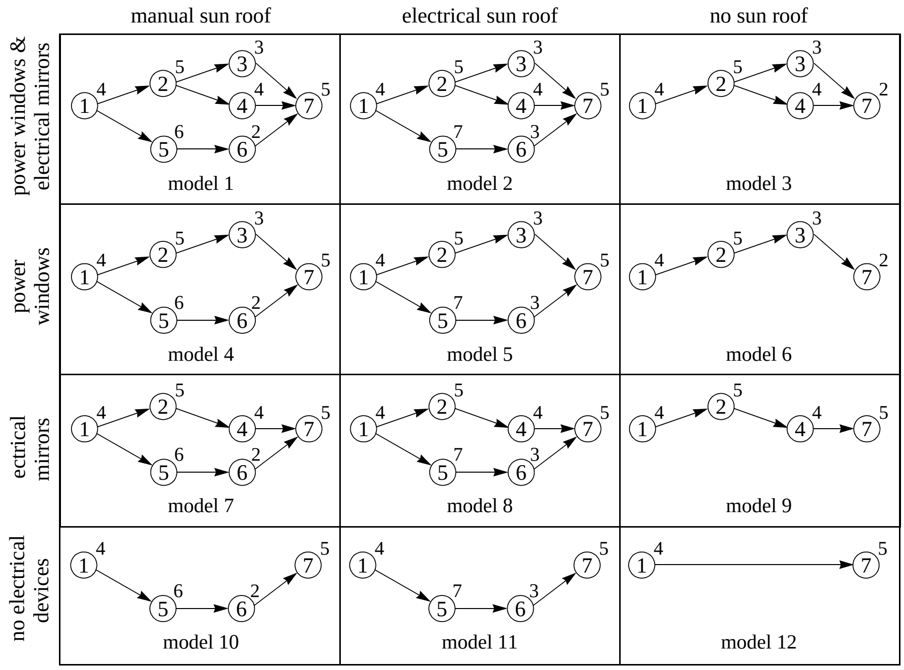

Figure 4: Precedence graphs of all models

used in a nested manner such that several XOR-nodes can be contained in a path.

4 Relationship between model-based and option-based approach

In the preceding section we have proposed a new approach for computing the joint precedence graph necessary for modeling and solving the mixed-model assembly line balancing problem. In this section, we demonstrate that both, the traditional and the new approach, really end up with the same joint precedence graph, i.e., it is shown that the new approach works correctly from a theoretical point of view. Furthermore, we argue that the option-based approach is much less expensive with respect to the number of estimations required and, thus, the better choice in practice.

In order to compare both approaches, consider Figure 4 which presents all models for the example problem. Already this comparatively small example can be used to depict the reduction in the number of probability (and task time) estimates required when compared to the traditional model-based procedure. The option-mix requires estimates of 5 probabilities (the probabilities of the null-options must not be estimated explicitly) as opposed to the necessary 11 estimates of the model-mix, one for each possible model except one.

Because the number of possible models grows exponentially when the number of options is increased, the gap will be dramatic for real-world problem situations (cf. Table 1).

model	1	2	3	4	5	6	7	8	9	10	11	12
model-based	0.00	0.00	0.10	0.10	0.10	0.00	0.00	0.00	0.40	0.10	0.20	0.00
option-based	0.02	0.03	0.05	0.04	0.06	0.10	0.08	0.12	0.20	0.06	0.09	0.15

Table 3: Probability estimations for the 12 models

This is particularly true, whenever the number of multi-option tasks is relatively small as is usually the case in practice.

A closer investigation of the difference between both approaches reveals that the model-based procedure can be seen as the *worst case scenario* of the option-based approach, in the sense that the required effort for data collection and probability estimations of the option-mix will never exceed the effort required for determining the model-mix. Notice that any model can in fact be expressed as a virtual option obtained by feasibly combining *all* original options. Then assume that any task $i \in V$ is a multiple-option task (i.e., $V^C = V$) with option sets $O_i = O$ but task times $t_i(v)$ that are unequal for any pair of virtual options $v \in \mathbb{P}(O_i)$. Only in such unrealistic cases, the option-based approach requires the same number of estimations as the model-based one. In real-world assembly systems the number of virtual options v for which probabilities have to be estimated explicitly will be considerably lower than the number of models.

The *best case* is given if no multiple-option tasks exist. Then, the probabilities p_o for isolated occurrence of the options $o \in O$ are sufficient. Notice that this is true even if sales dependencies between options might exist. Though not being always indicated, the best case is frequently assumed in practice (see Section 5 for an in-depth analysis of this relaxed approach).

In the following, we show that the option-based approach generates the same joint precedence graph as the model-based one despite of the drastically reduced information basis.

First, we reconsider the example and compute the probabilities P_m for all models $m = 1, ..., 12$ (cf. Figure 4) by counting their proportionate occurrences in the sales database (cf. Table 2). Using these probabilities given in Table 3 to compute the joint task times $\bar{t}_i$ by applying equation (2) results in the same joint graph as depicted in Figure 2(b).

Instead of directly determining the model-mix on the basis of observed frequencies, we can also employ the option-mix estimates to determine model probabilities P_m under the (not necessarily realistic) assumption that all (virtual) options which are not related to the same set of tasks occur independently of each other (second row of Table 3). The latter values are obtained by multiplying the probabilities of (virtual) options, whereas the former values are based on real sales data and thus can reflect customer preferences. Which estimate of model probabilities actually has a higher predictive power depends mainly on the quality and size of the sales database. Concerning the excessively large number of models, only a small part of which have ever been sold before, however, many P_m-values will be 0 which is avoided by the option-based prognosis.

However, independent of the accuracy of model probabilities, the joint task times result to the same values in both cases:

1. For each task $i \in V^A$ this is obviously true, because the task time is the same for all options and models, respectively.

2. The tasks $i \in V^B$ repeatedly occur in just those $|O_i|$ constellations as considered in formula (5) systematically spread over the models. So, the only difference consists in the probabilities p_o being split up over the models. In the sales data base of our example, the options 1 (manual sunroof) and 2 (electrical sunroof) occur for two and three times, respectively. These occurrences are directly transferred into probabilities $p_1 = 0.2$ and $p_2 = 0.3$. Option 1 is included in the models 1, 4, 7, and 10 (cf. Figure 4), option 2 in the models 2, 5, 8, and 11. Obviously, we get $P_1 + P_4 + P_7 + P_{10} = p_1$ and $P_2 + P_5 + P_8 + P_{11} = p_2$ independent of the distribution of occurrences within the respective model groups.

3. The same argument holds for tasks $i \in V^C$ based on virtual options which also represent the only constellations possible for the contained options within the models. In our example, task 2 appears in the models 1 to 3 as in the virtual option $\{3, 4\}$, in the models 4 to 6 as in the virtual option $\{3\}$, and in the models 7 to 9 as in the virtual option $\{4\}$. The joint probabilities of the model groups coincide with the probabilities of the respective virtual options.

The option-mix approach thus shifts the focus on those option dependencies which actually affect the production process and ignores all other interactions thereby saving unnecessary effort. This is especially useful if new options are introduced for which no reliable sales data exist.

5 On the importance of estimating joint option probabilities

The previous sections have demonstrated that there is no realistic alternative to using an option-based approach for constructing a joint precedence graph whenever the variety of models is considerable as is true for many consumer products. The effort of collecting information on tasks, precedence relations and occurrence probabilities is drastically reduced. Nevertheless, there remains considerable effort in collecting and computing data. The more multiple-option tasks exist, the more virtual options have to be added and the more probabilities and task times have to be estimated.

The authors' experience has shown that at major German automobile manufacturers, the joint probabilities are often not accounted for properly, because they are generally not believed to have an impact on planning results large enough to justify the increased effort. By doing so, the processing times of multiple-option tasks are systematically overestimated, so that the resulting line balances tend to require excessive resources, e.g. additional stations, compared to an ALBP-solution based on a proper calculation of task times.

In the following, a computational experiment is conducted, which quantifies the risk of waste and provides more detailed insights on the relationship between joint option occurrences and resource utilization. In order not to bias the results of the study by too many influencing factors, we employ a straightforward experimental design which focuses on the core aspect of option dependencies. If already a small number of those dependencies leads to excessive stations, a closer investigation of the trade-off between planning effort and the quality of resulting line balances is gratuitous, as the cost of additional stations will typically exceed planning cost by far.

The basic idea is to compare the solution quality of two (in other respects identical) ALBP-instances at a time, one with a proper estimation of multiple-option tasks' processing times (procedure 1) and one which overestimates processing times by neglecting joint occurrences of options (procedure 2). This way, excessive resources induced by procedure 2 can be determined. The extent of waste is mainly affected by two influencing variables, each of which is systematically varied as a simulation parameter:

- *Number of multiple-option tasks:* It is supposed that the higher the number of multiple-option tasks, the higher the amount of excessive resources. To account for this effect, the simulation parameter $\eta \in \{0.05 \cdot i | i = 1, ..., 10\}$ is introduced, which denotes the expected fraction of multiple-option tasks.

- *Probability of joint occurrence:* It is further expected that the higher the probability of joint occurrences of independent options, which share a multiple-option task, the more the processing time is overestimated and, thus, waste of resources is enhanced. The simulation parameter $\psi \in \{0.1 \cdot i | i = 1, ..., 9\}$ is used to calculate the probability of the joint appearance of (two) options in comparison to their separate occurrences.

In a full-factorial experiment, all values of the simulation parameters are combined with each other, so that in total 90 test cases are generated. For each of these test cases we solve several ALBP-instances and compare procedures 1 and 2. We employ the well-established 64 ALBP-instances of Talbot et al. (1986) for SALBP-1. It is assumed that the original problem instances of Talbot's data set are the result of a proper estimation of task times provided by procedure 1. For each of the instances a modified version, which represents the overestimation of task times by procedure 2, is generated as follows:

(1) Draw an uniformly distributed random number R_i for each task $i \in V$. If $R_i \leq \eta$ then task i is assumed to be a multiple-option task with two individual options not relevant for any other multi-option task. For the sake of convenience, these options are given the numbers 1 and 2 in each case.

(2) For each multiple-option task i calculate
   (a) the probabilities of the options' separate occurrences p_1 and p_2 by drawing uniformly distributed random numbers out of the interval $[0.1; 0.9]$,
   (b) the probability of the options' joint occurrence:
   $p(\{1,2\}) = \psi \cdot min\{p_1, p_2\}$, and
   (c) the overestimated task time $\hat{t}_i$ which is based on the correct (integral) task time t_i of the original instance in Talbot's data set as follows:
   $\hat{t}_i = Round(t_i \cdot \frac{p_1+p_2}{p_1+p_2-p(\{1,2\})})$

The overestimated task time $\hat{t}_i$ is rounded to the nearest integer value, because the employed solution procedure for ALBP presupposes integer value task times which is common in ALB research (e.g. Baybars, 1986).

The modified problem instance is solved to optimality with the branch and bound approach SALOME developed by Scholl and Klein (1997, 1999). This result is compared to the known optimal objective value of the original test instance. In this fashion the total $90 \cdot 64 = 5760$ problem instances are generated and solved.

The results of this computational experiment are summarized in Table 4 and visualized in Figure 5. As Figure 5 shows, already few multiple-option tasks with low probabilities for joint option occurrences may result in excessive stations. With $\eta = 0.05$ and $\psi = 0.1$ in 2 out of 64 test instances an excessive station occurred, which results in an average relative deviation of 1%. As expected, the risk of wasted resources increases, the higher the number of multiple-option tasks and the higher the probabilities of joint occurrences become. Deviations ascend more or less in a linear manner with increasing simulation parameters η and ψ, until at $\eta = 0.5$ and $\psi = 0.9$ remarkable 57 of 64 test instances show at least one additional station with an average relative deviation of 23% and a maximum of 5 additional stations.

In light of the strategic nature of assembly line balancing, already the saving of a single station can result to a considerable reduction in cost, as its installation not only entails investments in additional machines and assembly conveyors, but also increases the length of the line and thus results in higher work-in-process. In contrast to that the forecasting effort can be reduced to simple database queries, whenever the historical sales database is sufficient in size. Even if the base model is in an early stage of its life-cycle, it is common practice to fall back on sales data of anterior models. The experimental design is thus sufficient to demonstrate the importance of estimating joint option probabilities in the option-mix.

6 Conclusion

This paper proposes a modified approach for the generation of joint precedence graphs in order to balance mixed-model assembly lines. The modified approach is based on option-mix forecasts and is without alternative whenever the product variety is too high to allow for reliable model-mix forecasts, as it reduces the investigation to those task-option interdependencies which actually affect production planning. Nevertheless, it was also shown that completely neglecting all option dependencies bears the considerable risk of systematically overestimating task times and resource utilization which might in turn lead to an excessive waste of resources. The presented approach can thus be seen as the ideal compromise of reducing planning effort as much as possible without sacrificing the necessary precision and is thus highly recommendable for practical applications.

η	ψ 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	Total
0.05	1/0.03/1	2/0.08/1	4/0.13/1	3/0.14/1	3/0.14/1	4/0.16/1	4/0.14/1	6/0.25/1	6/0.25/1	4/0.15/1
0.1	3/0.11/1	3/0.13/1	4/0.14/1	3/0.14/1	5/0.22/1	7/0.3/1	6/0.3/1	6/0.3/2	7/0.38/2	5/0.22/2
0.15	2/0.08/1	3/0.14/1	5/0.2/1	6/0.25/1	5/0.27/1	7/0.34/1	9/0.42/2	8/0.55/2	10/0.53/2	6/0.31/2
0.2	3/0.11/1	4/0.19/1	7/0.28/1	6/0.25/1	8/0.38/1	9/0.48/1	9/0.52/2	10/0.61/3	12/0.67/2	7/0.39/3
0.25	3/0.13/1	5/0.22/1	7/0.28/1	7/0.33/1	9/0.5/1	9/0.5/2	11/0.72/2	14/0.88/3	12/0.84/3	9/0.49/3
0.3	4/0.16/1	6/0.22/1	7/0.31/1	8/0.39/1	9/0.48/2	12/0.69/2	12/0.77/2	13/0.86/2	17/1.11/3	10/0.55/3
0.35	3/0.13/1	5/0.20/1	7/0.33/1	9/0.47/1	10/0.58/2	11/0.67/2	13/0.86/2	13/0.91/3	19/1.33/3	10/0.61/3
0.4	4/0.16/1	5/0.23/1	7/0.39/1	11/0.58/1	12/0.73/2	13/0.78/3	15/1/4	15/1.19/4	21/1.41/4	12/0.72/4
0.45	4/0.16/1	6/0.23/1	9/0.47/1	10/0.56/2	13/0.81/2	14/0.86/3	16/1.08/3	19/1.27/3	22/1.47/4	12/0.77/4
0.5	4/0.16/1	6/0.27/1	9/0.5/1	10/0.58/2	12/0.83/2	14/0.91/3	18/1.19/3	20/1.41/4	23/1.64/5	13/0.83/5
Total	3/0.12/1	4/0.19/1	7/0.3/1	7/0.37/2	9/0.49/2	10/0.57/3	11/0.7/4	13/0.82/4	15/0.96/5	9/0.5/5

average relative deviation [percent]/average absolute deviation [stations]/maximum absolute deviation [stations]

Table 4: Results of the computational experiment

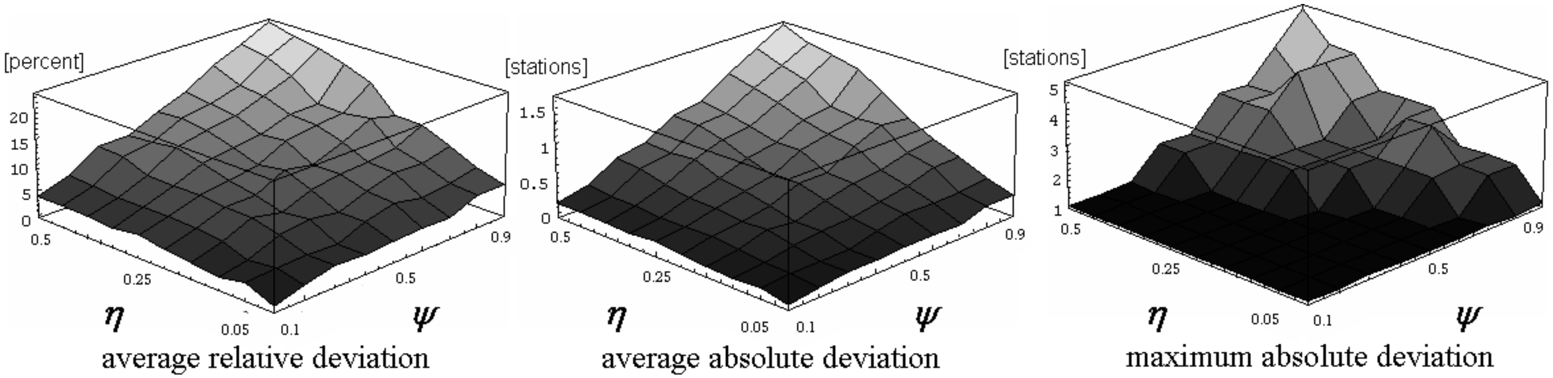

Figure 5: Results of the computational experiment

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